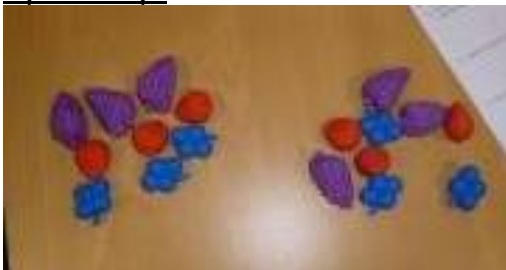



| Fractions Progression | |
|---|---|
| Objectives/Activities | Photos |
| <p>Year R - Key Vocabulary -fair, sharing, the same, different, whole, equal, altogether, double, half.</p> <p>1. Understanding 'fairness' Finding equal and unequal groups. Realising when two groups have the same, they are equal.</p> <p>Possible activities and opportunities:</p> <ul style="list-style-type: none"> • During snack time... give one child 2 bananas, another 2 apples - ask 'is this fair?' Discuss. • During play... rewards for 'sharing fairly' e.g. 'we have got 4 toy cars and 2 children, what should we do?' Extend by questioning - what if there were 3 children? <p>2. Sorting fairly (link to Venn diagrams) Possible activities:</p> <ul style="list-style-type: none"> • Use manipulatives such as dinosaurs, bears etc to sort - develop consideration of colour, size, type of toy etc. • Have hoops and characters in charge of each hoop e.g. two dinosaurs - each dinosaur needs the same as the other in their hoop - begin with an even number of the same coloured unifix cubes - then introduce another set of a different colour - how are they going to make sure the dinosaurs still have equal sets of objects in terms of colour <i>and</i> number? Extend by introducing a wider range of colours, numbers and types of objects. • Sorting the bean bags during P.E so that it is fair for each group. <p>3. Doubling Possible activities:</p> <ul style="list-style-type: none"> • One hand of five fingers and another hand of 5 fingers make 10 fingers! • 2 rows of eggs in boxes (simple representations of arrays) • Numicon (one piece of 2 and another piece of 2 makes 4 altogether -overlay on a piece of four) • Lines of counters (or 'magic beans'), they | <p>Equal Groups</p>  <p>Here two equal groups have been made where the child had to consider colour, number and type of object.</p> <p>Doubling using numicon</p>  <p>Here the child has been asked to double 1, 4 and 5. They understood that to double meant "the same again".</p> |

need to recognise that to double a number or a set of objects you simply need the same amount or type of objects again - one row of 6 'magic beans' provided to the child. If they make another *identical* row, they are doubling!

Year 1 -

Key Vocabulary: whole, equal, parts, half, quarter

1. Children need the concept 'parts of a whole'. The word and notion of a 'whole' must be explored extensively early on.

- Opportunities for this daily: 'Can the whole class stand up?'
- 'The whole of this group can go to play.'
- 'The whole pot of pencils must be sharpened.'
- The whole group of children on this table is 4 but the whole of the other table is 6!

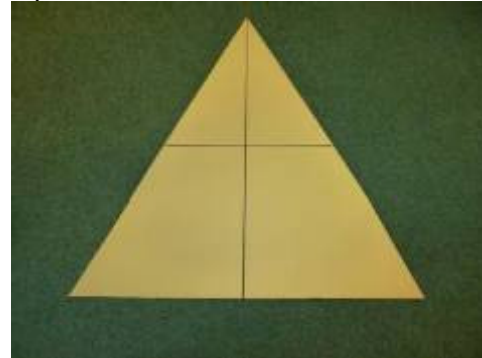
2. Recognise, find and name a half as one of two equal parts of an object, shape or quantity.

*when splitting objects in half always have another 'whole' to compare the parts to.

Possible activities:

*always use as many representations / manipulatives as possible e.g. don't just show pizza / circles.

- Use shapes (it is important to use irregular shapes) to fold and cut to find two equal parts and identify one part as a half. (Children should be folding and cutting the shapes to fully explore equality. Simply drawing or shading can lead to the misconception that '2 parts = half' regardless of size.)
- Use counters or sorting bears etc, to sort into two equal groups. It is vital that the children consider and discuss number, size, colour and type of object before stating that they have half. (Begin to organise manipulatives into arrays to make visual links with multiplication and division and encourage systematic organisation.) Show arrangements of manipulatives which are organised in equal groups according to number but not size or colour - check understanding by asking the child why the arrangement in front of them is not equal?
- Food! (Try to use foods not stereotypically linked with fractions) e.g. strawberry laces will allow links later in progression with length and number lines.
- Numicon - use even numbered numicon

Equal or not?

Explain that some treasure has been found and it is to be split into four pieces as shown above. At this point some children may believe that the triangle has been split into quarters. Give four children a piece each and ask if they are happy? This will provoke discussion about fractions as equal parts as the children with smaller pieces are unlikely to be satisfied!

Equal or not?

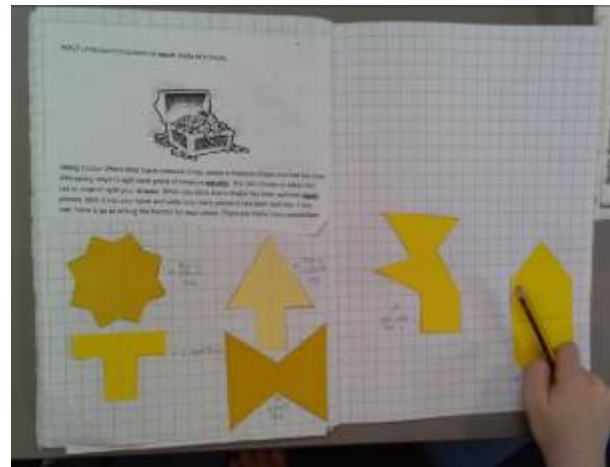
Many children will be happy that this group of 20 counters have been shared equally because there are 10 in each group! Use this as an opportunity to explore size, shape and colour!

pieces and pegs. 'How many ways can you show half?' (This will allow you to address or avoid the misconception that 'half' must be symmetrical.)

- Above activity can also be done with Cuisenaire - finding which colours show a half of a longer coloured rod.

3. Extend all of the above activities to find four equal parts to find quarters. Introduce the idea of half and half again to find quarters. *all the time explicitly teaching that a quarter is one part of the four.

Fractions of Shapes:



Allow children to explore unusual and irregular shapes by folding drawing and cutting.

How many ways can you show half?

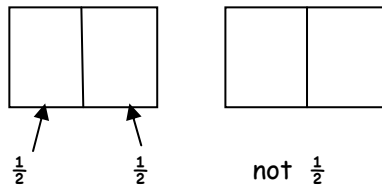


Children often believe that it is only half if the arrangement is symmetrical. This activity allows them to explore 'all possibilities' skills and develop a deeper understanding of half. It is also useful to get them to explore this task on different size numicon.

Year 2

Vocabulary: parts, out of, third.

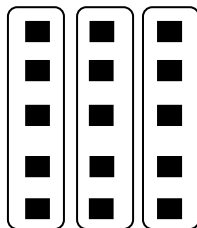
1. Begin by consolidating year 1 content and ensure year 1 vocabulary is secure before moving on.
2. Introduce notation - explicitly teach how to write $\frac{1}{2}$ and $\frac{1}{4}$ now that they fully understand what they are. Tip: make sure they know



Again manipulatives are needed.

3. Introduce a wider range of fractions - e.g. 3 equal parts = 3 thirds - leads into language of 'third'
 - Using Arrays to find the fraction of a set of objects.

If we want to find a third $\frac{1}{3}$ we split into 3 equal groups.



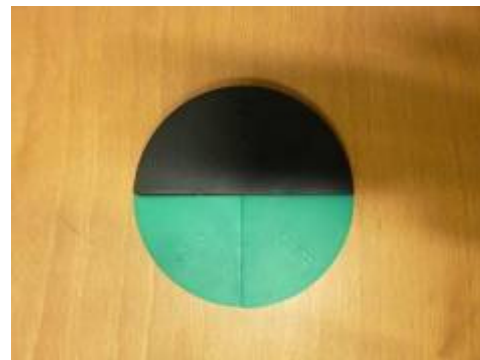
4. Non-unitary fractions. Look at $\frac{1}{4}$ and $\frac{3}{4}$ ($\frac{3}{4}$ should be the first non-unitary fraction) showing that they have 3 equal parts out of the 4 that make the 'whole'.
 - String activity on a line with notation for those that are ready. (Pick up individual pieces at different places on your line - ask 'how many parts do I have now?')
 - Counters/beads etc.
 - How many ways can you show $\frac{3}{4}$? E.g. using numicon - pegs in all holes, $\frac{1}{4}$ in one colour, $\frac{3}{4}$ in another - swap position of counters.
5. Count in fractions on a number line. (introduce through verbal counting forwards and back - e.g. in carpet count downs etc)
 - String on a number line
 - What would come next? Activities, and

Finding fractions using manipulatives - arrays



The child has split their beads into thirds, making sure that each group is equal in terms of number, size and colour. They have arranged their beads into lines. The child can now discuss a variety of fractions content including "what is one third of this whole?" and so on.

Using Cuisenaire to compare fractions and begin to understand that some are equivalent to others!



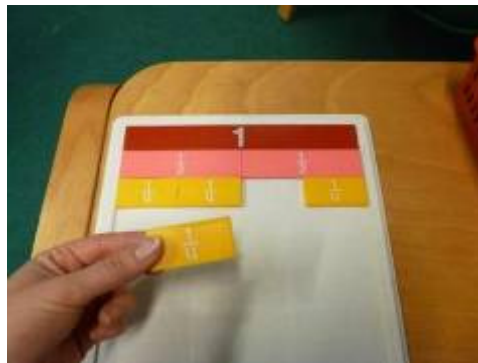
missing numbers on a line. (Do these regularly as mental starters for a range of fractions.)

6. Equivalent fractions.

Possible activities:

- Ribbon strips on a blank line - stick amounts down and compare. 'What do you notice?' 'Oh $2/4$ is the same as $1/2$!' extend - 'what else is the same as a half?' Children become curious at splitting string again and again to find many equivalent fractions.
- Stacking fractions using different shapes
- Physical fraction wall - that they can manipulate (not just a picture to notate / colour) - can also be made with cake - use log rolls so that the children don't become reliant on the circular representation of fractions.

Fraction wall that can be manipulated



Shape stackers are split into different amounts, the children can layer and compare!



Numicon can also be used for layering fractions to explore when they are equal!

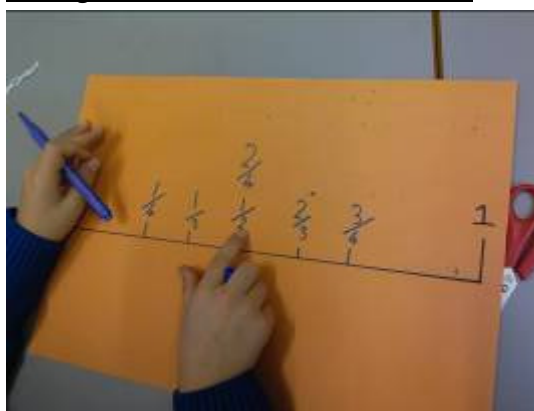


Year 3:

1. Start by ensuring understanding of 'equal' is secure.
2. Recognise, find and write fractions of a discrete set of objects.
Example activities:
 - Start simple - find half of these counters, bears, unifix etc. $\frac{1}{2}$ of 6 = 3
 - Then reinforce the link to finding quarter by halving again.
 - Next move on to using unifix or counters... split these into thirds - how many in one third? Etc.
 - Open ended tasks. E.g. Chocolate task pictured opposite.
 Leads naturally to arrays by organising manipulatives into arrays - seamless link to multiplication and division.
3. Recognise and use fractions as numbers.
Good opportunity to link to measures and numberlines.
 - Starter idea: Place fractions on a number line and count back and forward using different starting places. Vary the fractions used including same numerator and then fractions with different denominators.
 - Measure height of friends / tables / wood etc. Find fractions of height, length. Link to capacity - $\frac{1}{3}$ of your drink is?
4. Adding and subtracting fractions.
 - Use a variety of resources to combine fractions.
 - E.g. String - split into quarters, pick up 2 quarters and say 'one quarter and another quarter equals two quarters.' Move onto how to write it and, explore others, and subtraction.
 - Paper strips - fold then cut; label each part with the fraction. Then stick parts you are adding into book to see the total altogether. Also do this with subtraction - cross out how many parts you are subtracting.
5. Compare and order fractions with the same

Finding fractions of amounts using a chocolate bar:

Children above were given a chocolate bar and asked to start by deciding what the whole is. They then worked in pairs to find as many fractions of the bar as they could. Some children used arrays, some worked systematically - all used mathematical discussion about fractions! Vary the task by choosing the size of the whole according to ability (6 by 4 chocolate bar is good as it allows for lots of fractions to be found!)

Placing Fractions on a Number Line:

Use ribbon to find fractions between nought and one by folding and cutting then placing in order to notate on a blank number line.

denominator, and with different denominators.

- Food can be a powerful tool for this! 'He's got more than me!' type discussions.

6. Equivalence

- Use shape stackers to explore 'which ones are the same?' open ended activities. Using both square *and* circular manipulatives.
- Return to ribbon or squared strips of paper on a number line for direct comparison - sticking pieces down each time.
- Manipulative fraction wall - avoid using traditional fraction walls where you simply colour and label. Ensure children are building / moving a form of the wall. This way when you question them, they can show you their understanding by layering or placing fractions next to each other to prove equivalence.
- Challenge: Prove using play dough and scales that $\frac{2}{4} = \frac{1}{2}$, extend to also include objectives related to weight and reading scales - use a variety of physical weights and a variety of scales (traditional and electronic) to prove that a certain fraction is equal to another.

Using numicon to add and subtract fractions



Above the child has written:

$$\frac{2}{9} + \frac{6}{9} = \frac{8}{9}$$



Above the child has written:

$$\frac{6}{6} - \frac{2}{6} = \frac{4}{6}$$

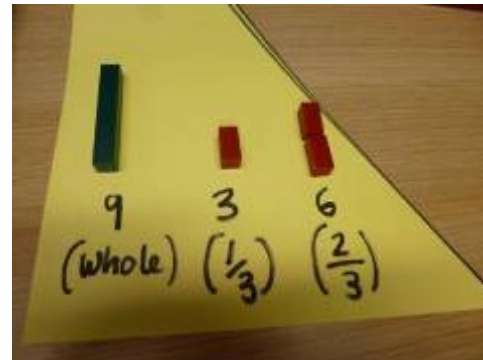
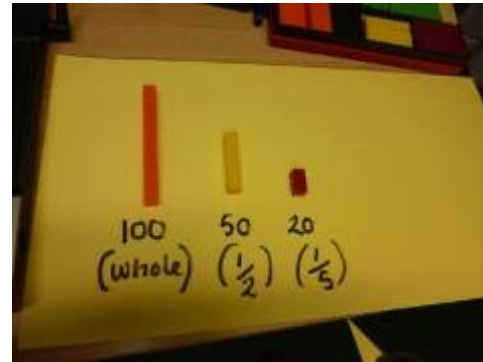
Year 4:

(consolidation of Year 3 objectives is vital in order to avoid potential misconceptions)

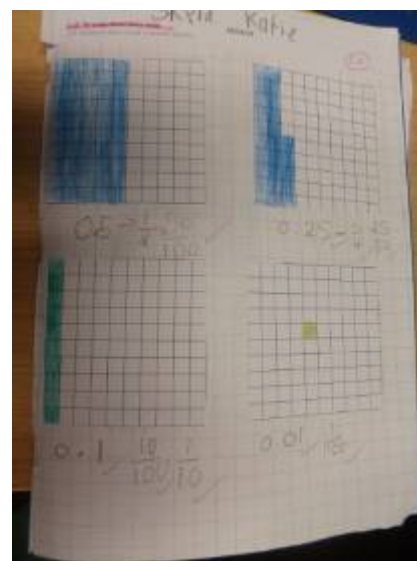
1. Add and subtract fractions with the same denominator - see year 3 objective.
2. Equivalent fraction families.
 - 'What other fractions = $\frac{1}{2}$?' leads into simplifying fractions where appropriate.
 - See year 3 activities for equivalence but change the focus to finding fraction families - Make fraction family mind maps for display in the classroom (Start easy - $\frac{1}{2}$ in the middle and lots of fractions equal to half around the outside - differentiate by asking more able children to do a mind map of fractions equal to $\frac{3}{4}$ etc. It is vital that they use a manipulative whilst doing this task to show their conceptual understanding as well as their understanding of the number relationship. Continually ask them to prove that one of their chosen fractions is equal to the number in the middle of their mind map).
3. Recognise and write decimal equivalence of tenths and hundredths.
 - Link to real life - money
 - Explore hundredths through shading empty 100 squares (also link to finding $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$ as decimals.)
 - Use place value sliders when dividing any number (1 or 2 digit) by 10 or 100.
 - Link to measures in order to develop understanding of decimals - do they understand the difference between 1.2m and 1.02m?

Decimal objectives

- recognise and write decimal equivalents to
 - $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$
- find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as units, tenths and hundredths
- round decimals with one decimal place to the nearest whole number
- compare numbers with the same number of decimal places up to two decimal places

Cuisenaire

Children select one of the rods and choose a whole number for it to represent. They then explore the other rods and record the fractions that they represent. This can be extended to also include decimals.

Exploring the relationship between fractions and decimals using blank 100 squares

The children recognised that shading 25 squares out of 100 can be written as 25/100. This is the

same as 0.25. Many children then notice or discuss that this is also the same as $\frac{1}{4}$. This activity allows for further exploration of the fact that fractions do not have to be uniform or symmetrical (they can colour any square within the whole).

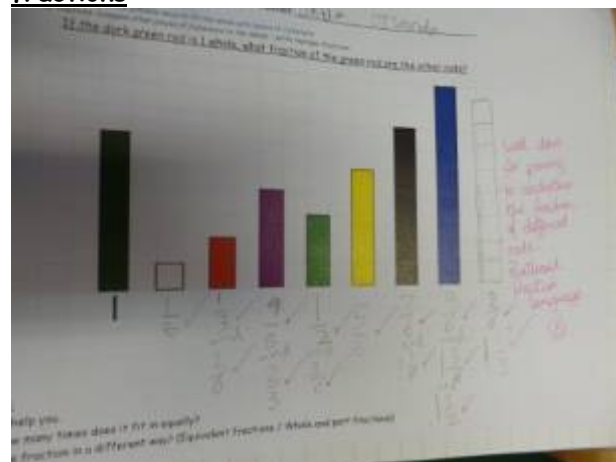
Place value slider



Place value sliders allow the children to see division as scaling the numbers down.

Year 5:

1. Recognise mixed numbers and improper fractions and convert from one to the other.
 - Introduce this concept in a really basic way - Show a whole of an object (e.g. a chocolate bar or a piece of paper). Show another whole, split the second whole in half. Show one whole and one half together and ask what you have got (one and a half!). Extend by teaching that we write this as $1 + \frac{1}{2} = 1 \frac{1}{2}$
 - Numicon stacking to build up whole and further fractions.
 - Use ribbon to build a numberline like in previous years but at this stage they should be working with fractions numberlines that go beyond 1.
 - Cuisenaire is an extremely useful resource at this stage.
2. Adding and subtracting fractions with the same denominator to include improper fractions and convert to mixed numbers e.g. $\frac{6}{5} = 1 \frac{1}{5}$
 - Use same equipment as above to add and subtract using manipulatives.
3. Compare and order fractions whose denominators are all multiples of the same number.
 - Look at the multiplicative nature of fractions. E.g. $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$ (all can be divided by 2 to get you back to $\frac{1}{2}$, however all represent the same amount.) Practical examples should be used to prove this so that children do not have the misconception $\frac{4}{8}$ is more than $\frac{1}{2}$.
4. Add and subtract with denominators that are multiples of the same number e.g. $\frac{2}{4} + \frac{4}{8} = ?$
5. Show multiplication of fractions visually and using diagrams, models.
 - Array grid to show split of the first fraction, then split the other way by the other fraction - where it overlaps is the answer out of the entire whole.
 - Food examples - pizza.
6. Understand division with remainders written as fractions. E.g. $15 \div 2 = 7 \frac{1}{2}$ This can then

Using Cuisenaire to discover improper and proper fractions

Children were given the whole then had to solve what fractions the other rods represent. This lead naturally onto children looking at fractions beyond whole. The notation of this can then be explored.

Numicon

With the whole as ten, each peg represents $\frac{1}{10}$ (0.1). Children can then stack up the numicon to show any decimal number. For example the picture above shows 1.3 or $1 \frac{3}{10}$ or $\frac{13}{10}$.

You can then use the same model to add and subtract tenths to solve fraction problems above 1 whole.

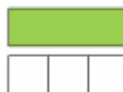
be linked to understanding of fractions. E.g. half as a decimal = 0.5 so $15 \div 2 = 7.5$

Decimal and percentage objectives:

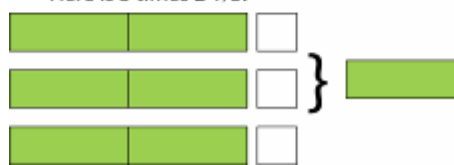
- read and write decimal numbers as fractions (e.g. $0.71 = \frac{71}{100}$)
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- round decimals with two decimal places to the nearest whole number and to one decimal place
- read, write, order and compare numbers with up to three decimal places
- solve problems involving number up to three decimal places
- recognise the per cent symbol (%) and understand that per cent relates to "number of parts per hundred", and write percentages as a fraction with denominator hundred, and as a decimal fraction
- solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those with a denominator of a multiple of 10 or 25.

Multiplying an improper fraction by a whole number

Say the light green rod represents 1.
Then the white rod represents $\frac{1}{3}$.

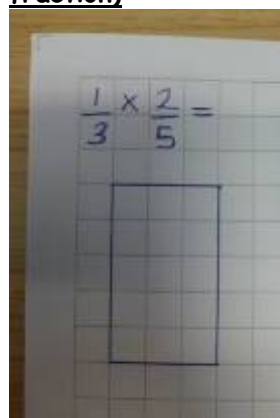


Here is 3 times $2 \frac{1}{3}$.

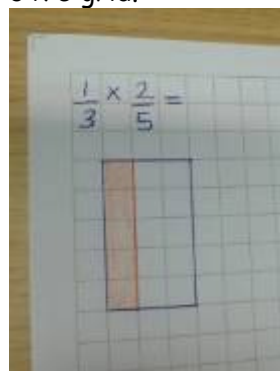


So the answer = 7 as the 3 white rods = 1 whole

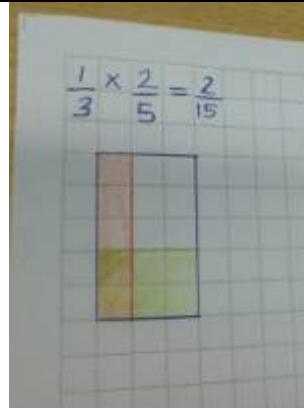
Multiplying a fraction by another fraction using arrays (the method shown below allows children to understand visually the multiplication of fractions - you are finding a fraction of another fraction)



Step 1: Work out the 'whole' by drawing an array using the 2 denominators. So here we have drawn a 3 x 5 grid.



Step 2: Shade the first fraction of the whole. Here we have shaded $\frac{1}{3}$ in red.



Step 3: Shade the second fraction of the whole. Here we have shaded $\frac{2}{5}$ in green. The answer is where the two fractions overlap. Here 2 squares overlap out of 15.

Year 6:

1. Consolidate year 5 skills!
2. Begin to explore common factors in order to simplify fractions. (Use common multiples to express fractions with same denominator). This skill is vital for understanding all year 6 coverage and should be fully secure before moving on to next steps. When given a range of fractions can they simplify them and name the equivalent pairs?
3. Add and subtract fractions with different denominators (using skill 1 as a starting point!)
4. Multiply fractions and use a variety of images to communicate their understanding (revisit year 5 coverage where needed) and then convert answer to simplest form (again using skill 1!)
5. Divide proper fractions by whole numbers.

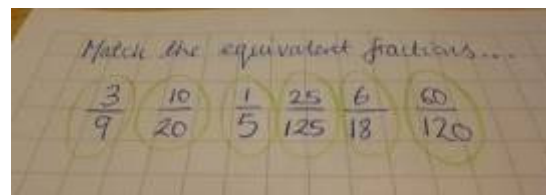
Decimal and percentage objectives:

- associate a fraction with division and calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. $\frac{3}{8}$)
- identify the value of each digit to three decimal places and multiply and divide numbers by 10, 100 and 1000 where the answers are up to three decimal places
- multiply one-digit numbers with up to two decimal places by whole numbers
- use written division methods in cases where the answer has up to two decimal places
- solve problems which require answers to be rounded to specified degrees of accuracy
- recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.

Ratio and proportion objectives:

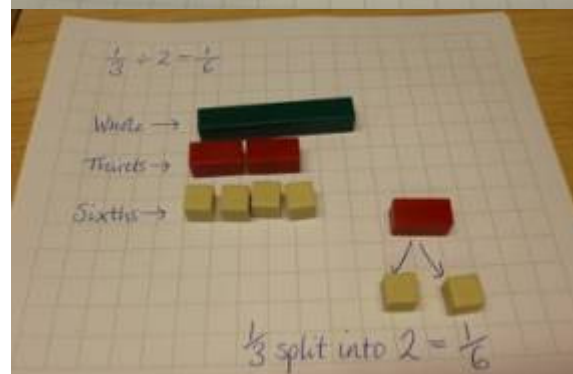
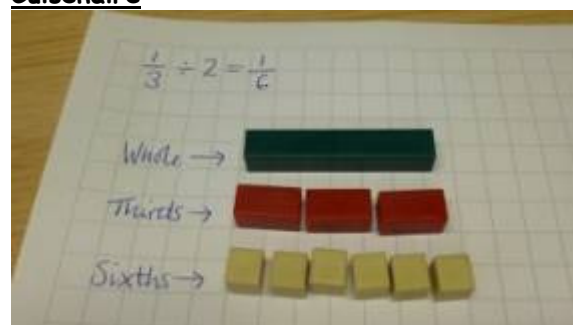
- Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts
- Solve problems involving the calculation of percentages (e.g. of measures) such as 15% of 360 and the use of percentages for comparison
- Solve problems involving similar shapes

Understanding fractions



Step 1: Simplify the fractions,
 Step 2: Match the equivalent fractions,
 Step 3: Prove the fractions are equivalent using a range of manipulatives.

Dividing a fraction by a whole number using Cuisenaire



To calculate $\frac{1}{3} \div 2 =$

Ensure the rods you choose to represent thirds are divisible by 2. Then separate $\frac{1}{3}$ away from the whole and divide by 2. Check what your final fraction represents by comparing back to the whole. So in this example it is $\frac{1}{6}$

Another example:



$\frac{2}{5} \div 6 =$

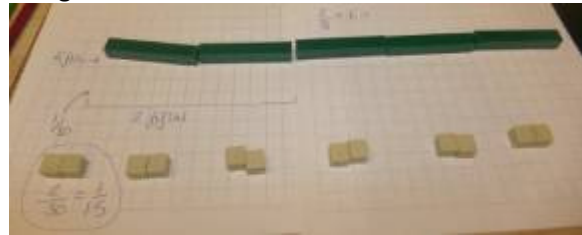
Ensure the rod you choose to represent $\frac{1}{5}$ is

where the scale factor is known or can be found

- Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples.

divisible by 6.

Take 2 of your fifths and split into 6 equal groups by using smaller rods.



Compare your final fraction to the whole to understand what the answer represents.

Adapt this task to use numicon equipment - make sure you choose the shape that is divisible by the whole number - then numicon pegs will allow you to manipulate your fractions.